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Double Diffusive Convection in a Viscoelastic Fluid Saturated rotating anisotropic porous layer with internal heat source

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Abstract-In this paper, the effect of internal heating on double diffusive convection in a rotating anisotropic porous medium saturated with a viscoelastic fluid,which is heated and salted from below,is studied analytically. Linear stability analysis has been performed by using Normal mode technique and nonlinear theory is based on minimal representation of Fourier series up to two terms. The modified Darcy model,which includes the time derivative and Coriolis terms has been employed in the momentum equation. The effects of Taylor number,solute Rayleigh number,internal heat source parameter, diffusivity ratio,relaxation and retardation parameters, thermal and mechanical anisotropy parameters on the stationary and oscillatory convection are obtained and shown graphically. Also, heat and mass transports have been obtained in terms of the Nusselt number and Sherwood number respectively and presented through Figs.

Index Terms-Viscoelastic fluid; Double diffusive convection; Rotation; Internal heat source; Porous media.

1. INTRODUCTION

Most of the studies in relevant area are mainly dealt with isotropic porous media; however there are many physical situations where thermal and mechanical anisotropy exists in porous matrix, one of such examples is our geothermal environment. Anisotropy is generally a consequence of preferential orientation of asymmetric geometry of porous matrix or fibers and is in fact encountered in numerous systems in industry and nature, also in artificial porous matrix anisotropy can be made deliberately according to applications. Srivastava et al. [5] studied the effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium.

There is large number of practical situations in which convection is driven by internal heat source. Internal heat generation arises in many important contexts, including reactor safety analyses, metal waste that is produced by spent nuclear fuel, fire and combustion studies, and the storage of radioactive materials. The study concerning internal heat source in porous media is provided by Tveitereid [27], performing thermal convection in a horizontal porous layer with internal heat source. Hill [3] performed linear and nonlinear analyses on the double-diffusive convection in a porous layer with a concentration based internal heat source. Bhadauria et al.[9] studied the effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium. Govender [14] investigated the Corioliseffect on the stability of centrifugally driven convection in a rotating anisotropic porous layer subject to gravity.

The studies of double diffusive convection in porous media plays very significant roles in many areas such as in petroleum industry, solidification of binary mixture, migration of solutes in water saturated soils. Other example includes geophysics system, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, Earth's oceans, magma chambers etc. The onset of thermal instability in a horizontal porous layer was first studied extensively by Horton and Rogers [15] and Lapwood [18]. However, Nield [28] was first to investigate double diffusive generalization of the Horton–Rogers–Lapwood problem, performing only linear stability analysis. Some other researchers who have worked on double diffusive convection in a porous medium are Taunton et al.[37], Patil and Vaidyanathan [31,32], Griffith [13]. The onset of double diffusive convection in a horizontal porous layer has been investigated by Rudraiah et al. [34] using a weak non-linear theory. The problem of double diffusive convection in a porous media has been presented by Ingham and Pop [16], Nield and Bejan [19] and Vafai [39,40], Vadasz[41,42]. The study was continued by Poulikakos [30], Travison and Bejan [38], Momou [22] etc.

The study of double diffusive convection in a rotating porous media is important due to both, its theoretical and practical applications in engineering. Some of the important areas of applications in engineering include the food and chemical process, solidification and centrifugal casting of metals, rotating machinery, petroleum industry and biomechanics problems. There are only few studies available on double diffusive convection in the presence of rotation. Chakrabarti and Gupta [4] have analyzed the nonlinear thermohaline convection in a rotating porous medium. The effect of rotation on linear and nonlinear double diffusive convection in a sparsely packed porous medium was studied by Rudraiah et.al. [34]. Malashetty et al. [23] studied the effect of rotation on the onset of double diffusive convection in a Darcy porous medium saturated with couple stress fluid. Malashetty and Heera [24, 25] studied the effect of rotation on the onset of double diffusive convection in a horizontal anisotropic porous layer. Gaikwad [12] have done the linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in presence of Soret effect. Sulochana et.al [10] studied the onset of double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer. Bhadauria et al. [7] studied cross diffusion convection in a Newtonian fluid-saturated rotating porous medium.

The work published on natural convection of viscoelastic fluids in porous media is fairly limited. Convection in a viscoelastic fluid-saturated sparsely packed porous layer is studied by Rudraiah et al. [33, 35]. Mardones et al. [20, 21] have investigated the Rayleigh-Benard convection for stationary convection in a binary viscoelastic fluid. Yoon et al. [43, 44], Kim et al. [17], and Bertola and Cafaro [6] studied the stability of a viscoelastic fluid where an existing constitutive model, which is rather simple, was employed to examine the effects of relaxation and retardation times on the stationary and oscillatory convection in a horizontal porous layer heated by a constant temperature. Park and Park [29] studied Rayleigh-Benard convection of viscoelastic fluids in arbitrary finite domains. Convective instabilities in a viscoelastic-fluid-saturated porous medium with throughflow have been studied by Shivakumara and Sureshkumar [36]. Linear and nonlinear stability analyses of thermal convection for Oldroyd-B fluids in a porous media heated from below has been studied by Zhang et al. [45]. Malashetty et al. [26] studied the onset of convection in a binary viscoelastic fluid-saturated porous layer. Kumar and Bhadauria [1] have studied non-linear two-dimensional double diffusive convection in a rotating porous layer saturated by a viscoelastic fluid. Gaikwad et al. [11] performed onset of Darcy-Brinkman convection in a binary viscoelastic fluid-saturated porous layer with internal heat source. Recently Srivastava et al. [2] have studied linear and weak nonlinear double diffusive convection in a viscoelastic fluid saturated anisotropic porous medium with internal heat source.

In the present literature, no work is available on double diffusive convection in a rotating porous layer saturated by a viscoelastic fluid with an internal heat source. Therefore, in the present study stability analysis of internal heating effect on double diffusive convection in a rotating anisotropic porous medium saturated with a viscoelastic fluid has been done.

2. GOVERNING EQUATION

Consider a viscoelastic fluid saturated porous medium, confined between two infinitely extended horizontal planes at $z=0$ and $z=d$, heated from below and cooled from above. Darcy model has been employed in the momentum equation. Further, an internal heat source term has been included in the energy equation. A Cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis as vertical upward. The system is rotating about z-axis with a constant angular velocity Ω. An adverse temperature gradient is applied across the porous layer and the lower and upper planes are kept at temperature $T_0+\Delta T$ and T_0, and concentration $S_0+\Delta S$ and S_0 respectively. The physical configuration of the model is reported in the Fig.A.

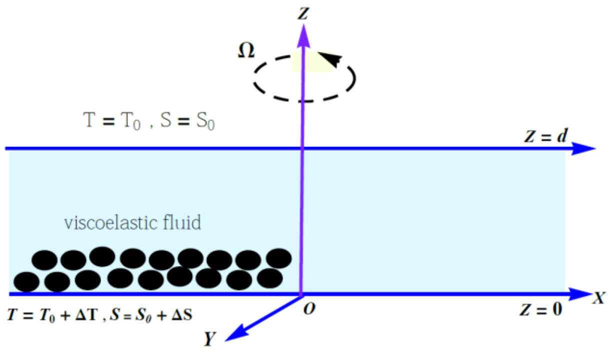


Fig .A: Physical configuration of the problem

$$\nabla . q=0 \tag{1}$$

$$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\frac{2\rho_0}{\phi}(\Omega\times q)+\frac{\mu}{\kappa}\left(1+\bar{\lambda}_2\frac{\partial}{\partial t}\right)=\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)$$
$$(-\nabla p+\rho_0 g) \tag{2}$$

$$\gamma\frac{\partial T}{\partial t}+(q\cdot\nabla)T=\nabla(\kappa_T\cdot\nabla T)+Q(T-T_0) \tag{3}$$

$$\phi\frac{\partial S}{\partial t}+(q\cdot\nabla)S=\kappa_S\nabla^2 S \tag{4}$$

$$\rho=\rho_0\left[1-\beta_T(T-T_0)-\beta_S(S-S_0)\right] \tag{5}$$

$$T=T_0+\Delta T;\ \text{at}\ z=0\ \text{and}\ T=T_0;\ \text{at}\ z=d; \tag{6}$$
$$S=S_0+\Delta S;\ \text{at}\ z=0\ \text{and}\ \mathrm{S}=S_0;\ \text{at}\ z=d;$$

2.1 Basic Solution

At this state, the velocity, pressure, temperature and density profiles are given by

$$q_b = 0,\ p = p_b(z),\ T = T_b(z),\ S = S_b(z),\ \rho = \rho_b(z). \quad (7)$$

Substituting Eq. (7) in Eq. (1-4), we get the following equations:

$$\frac{dp_b}{dz} = -\rho_b g, \quad (8)$$

$$\kappa_T \frac{d^2 T_b}{dz^2} + QT = 0, \quad (9)$$

$$\frac{d^2 S_b}{dz^2} = 0, \quad (10)$$

The solution of Eq. (9) and Eq. (10) subject to the boundary conditions (6), are given by

$$T_b = T_0 + \Delta T \frac{Sin\sqrt{R_i}\left(1-\frac{z}{d}\right)}{Sin\sqrt{R_i}}. \quad (11)$$

$$S_b = S_0 + \Delta S(1-\frac{z}{d}) \quad (12)$$

2.2 Perturbed Equation

Now, we superimpose finite amplitude perturbations on the basic state in the form:

$$q = q_b + q',\ T = T_b + T',\ p = p_b + p',\ S = S_b + S',\ \rho = \rho_b + \rho',\ (13)$$

and get the following equations

$$\nabla . q' = 0 \quad (14)$$

$$\left(1+\overline{\lambda}_1\frac{\partial}{\partial t}\right)\frac{2\rho_0}{\phi}(\Omega\times q') + \frac{\mu}{\kappa}\left(1+\overline{\lambda}_2\frac{\partial}{\partial t}\right)q' = -\left(1+\overline{\lambda}_1\frac{\partial}{\partial t}\right)$$
$$\left(\nabla p - \rho_0 g\left(\beta_T T' - \beta_S S'\right)\right) \quad (15)$$

$$\gamma\frac{\partial T'}{\partial t} + \left(q'\cdot\nabla\right)T' = \left(\kappa_{Tx}\cdot\nabla^2 T'\right) + QT'$$
$$-w'\frac{\partial T_b}{\partial z} \quad (16)$$

$$\frac{\partial S'}{\partial t} + \left(q'\cdot\nabla\right)S' + w'\frac{\partial S_b}{\partial z} = \kappa_S \nabla^2 S \quad (17)$$

$$\rho' = -\rho_0\left[\beta_T T' - \beta_S S'\right] \quad (18)$$

The resulting equations are non-dimensionalized using the following transformations;

$$(x', y', z') = (x^*, y^*, z^*)d,\ t' = t^*\left(\frac{d^2}{\kappa_{Tz}}\right),$$

$$q' = \frac{\kappa_{Tz}}{d}q^*,\ \lambda_1 = \frac{d^2}{\kappa_{Tz}}\lambda_1^*$$

$$\lambda_2 = \frac{d^2}{\kappa_{Tz}}\lambda_2^*,\ \ (u, v, w) = (u^*, v^*, w^*)\left(\frac{\kappa_{Tz}}{d}\right),$$

$$T' = (\Delta T)T^*,\ S' = (\Delta S)S^*,\ p' = \frac{\mu\ \kappa_{Tz}}{K_z}p^* \quad (19)$$

2.3 Non-Dimensionilized Equation

The non-dimensionalized equations (on dropping the asterisks for simplicity) are obtained as,

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\left(\kappa\times q'\right) + \left(1+\lambda_2\frac{\partial}{\partial t}\right)q_a + \left(1+\lambda_1\frac{\partial}{\partial t}\right)\nabla p = \left(1+\lambda_1\frac{\partial}{\partial t}\right)$$
$$\left(Ra_T T - \tau Ra_S S\right)k \quad (20)$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T_b}{\partial z} = \left(\eta\nabla_1^{\ 2} + \frac{\partial^2}{\partial z^2}\right) + R_i T \quad (21)$$

$$\frac{\partial S}{\partial t} + w\frac{\partial S_b}{\partial z} = \tau\ \nabla^2 S \quad (22)$$

where $q = \left(\frac{1}{\xi}u, \frac{1}{\xi}v, w\right)$ is anisotropic modified velocity vector, $Pr_D = \frac{\phi\gamma\nu d^2}{\kappa_T k}$ is Darcy-Prandtl number, $Ra_T = \frac{\beta_T g\Delta T K_z d}{\nu\kappa_{Tz}}$ is the thermal Rayleigh number, $Ra_S = \frac{\beta_S g\Delta S K_z d}{\nu\kappa_{Tz}}$ is the solute Rayleigh number, $R_i = \frac{Qd^2}{\kappa_{Tz}}$ is the internal Rayleigh parameter, $T_a = \left(\frac{2\Omega K_z}{\mu\phi}\right)^2$ is Taylor number, $\eta = \frac{\kappa_{Tx}}{\kappa_{TZ}}$ is thermal anisotropy parameter, $L_e = \frac{\kappa_{TZ}}{\kappa_S}$ is Lewis number, $\tau = \frac{\kappa_S}{\kappa_{Tz}}$ is diffusivity ratio, $S_r = \frac{K_{21}\Delta T}{\kappa_{TZ}\Delta S}$ is the Soret parameter, $\xi = \frac{K_x}{K_z}$ mechanical anisotropy parameter. The above system will be solved by considering stress free and isothermal boundary conditions as given below:

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \ \text{ on } \ z = 0, z = 1. \quad (23)$$

The pressure term from Eq. (20) is eliminated by taking curl of the momentum equation.

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)\frac{1}{\xi}\omega-\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\frac{\partial q}{\partial z}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)$$
$$\left[Ra_T\left(\frac{\partial T}{\partial y}i-\frac{\partial T}{\partial x}j\right)-\tau Ra_S\left(\frac{\partial S}{\partial y}i-\frac{\partial S}{\partial x}j\right)\right]$$

where $\omega=\nabla\times q$ is vorticity vector. On further taking curl, we get

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)Q-\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\frac{\partial \omega}{\partial z}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)$$
$$\left[Ra_T\nabla_1{}^2T-\tau Ra_S\nabla_1{}^2S\right] \tag{24}$$

where $Q=(Q_1,Q_2,Q_3)$,

$$Q_1=\frac{1}{\xi}\frac{\partial^2 v}{\partial y\partial x}+\frac{\partial^2 w}{\partial y\partial x}-\left(\frac{\partial^2 v}{\partial y^2}+\frac{1}{\xi}\frac{\partial^2 u}{\partial y^2}\right),$$

$$Q_2=\frac{1}{\xi}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 w}{\partial y\partial z}-\frac{1}{\xi}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right),$$

$$Q_3=-\left(\nabla_1{}^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w$$

3. LINEAR STABILITY ANALYSIS

Linear equations are

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)\frac{1}{\xi}\omega_z=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\frac{\partial w}{\partial z} \tag{25}$$

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\nabla_1{}^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w+\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\frac{\partial \omega_z}{\partial z}$$
$$=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[Ra_T\nabla_1{}^2T-\tau Ra_S\nabla_1{}^2S\right] \tag{26}$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla_1{}^2-\frac{\partial^2}{\partial z^2}-R_i\right)T=w \tag{27}$$

$$\left(\frac{\partial}{\partial t}-\tau\ \nabla^2\right)S=w \tag{28}$$

By eliminating T,S,ω_z from above equation, we obtain

$$\begin{bmatrix}\left(\frac{\partial}{\partial t}-\eta\nabla_1{}^2-\frac{\partial^2}{\partial z^2}-R_i\right)\left(\frac{\partial}{\partial t}-\tau\ \nabla^2\right)\left(1+\lambda_2\frac{\partial}{\partial t}\right)^2\\ \left(\nabla_1{}^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)+\xi T_a\left(1+\lambda_1\frac{\partial}{\partial t}\right)^2\frac{\partial^2}{\partial z^2}-\left(1+\lambda_1\frac{\partial}{\partial t}\right)\\ \left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\tau\ \nabla^2\right)Ra_T\nabla_1{}^2-\\ \left(\frac{\partial}{\partial t}-\eta\nabla_1{}^2-\frac{\partial^2}{\partial z^2}-R_i\right)\tau Ra_S\end{bmatrix}w=0 \tag{29}$$

$$w=\frac{\partial^2 w}{\partial z^2}=\frac{\partial^4 w}{\partial z^4}\ \text{ on }\ z=0,\ z=1.$$

Use normal mode technique

$$w=W(z)exp\left(i(lx+my)+\sigma t\right)\text{Sin}\,\pi z \tag{30}$$

where l, m are horizontal wave numbers and $\sigma=\sigma_r+i\sigma_i$, is the growth rate. Solve the above equation, the thermal Rayleigh number can be obtained as

$$Ra_T=\frac{(\sigma+\delta_2{}^2-R_i)(1+\lambda_2\sigma)\delta_1^2}{a^2(1+\lambda_1\sigma)}+\frac{T_a\pi^2\xi(1+\lambda_2\sigma)(\sigma+\delta_2{}^2-R_i)}{(1+\lambda_1\sigma)}$$
$$+\frac{(\sigma+\delta_2{}^2-R_i)}{(\sigma+\tau\delta^2)}\tau Ra_S \tag{31}$$

where $a^2=l^2+m^2$, $\delta^2=\pi^2+a^2$, $\delta_1{}^2=\frac{\pi^2}{\xi}+a^2$, $\delta_2{}^2=\pi^2+\eta a^2$. The growth rate σ is in general a complex quantity such that $\sigma=\sigma_r+i\ \sigma_i$. The system with $\sigma_r<0$ is always stable, while for $\sigma_r>0$ it will become unstable. For neutral stability state $\sigma_r=0$

3.1 Stationary State

Now we set $\sigma=0$ in Eq. (31) at the margin of stability. The expression of the thermal Rayleigh number for stationary mode of convection is found as given below:

$$Ra_T^{st}=\frac{(\sigma+\delta_2{}^2-R_i)\delta_1{}^2}{a^2}+\frac{T_a\pi^2\xi(\delta_2{}^2-R_i)}{a^2}$$
$$+\frac{(\delta_2{}^2-R_i)}{\tau\delta^2}\tau Ra_S \tag{32}$$

It is important to note that the critical wave number $a=a_c^{St}$ depends on the couple stress parameter and Taylor number. In the absence of Taylor number i.e. $T_a=0$ Eq. (32) gives

$$Ra_T^{st} = \frac{(\sigma+\delta_2{}^2-R_i)\delta_1{}^2}{a^2}+\frac{(\delta_2{}^2-R_i)}{\tau\ \delta^2}\tau Ra_S$$

For isotropic porous media when $\xi=\eta=1$ and the system without internal-heating, i.e., $R_i=0,$ we get

$$Ra_T^{st} = \frac{(\pi^2+a^2)^2}{a^2}+Ra_S$$

which is the result given by Malashetty et al. [25]. For single component fluid, $Ra_S=0,$ we obtain

$$Ra_T^{st} = \frac{(\pi^2+a^2)^2}{a^2} \qquad (33)$$

which has the critical value $Ra_T^{st}=4\pi^2$ for $a_c^{st}=\pi$ are the classical results obtained by Horton and Roger [16] and Lapwood [19] for single component fluid in porous layer.

3.2 Oscillatory State

For the oscillatory mode of convection, we set $\sigma=i\sigma_i$ in Eq. (27) and clear the complex quantities from the denominator, to obtain

$$Ra_T^{osc}=\Delta_1+i\sigma_i\Delta_2 \qquad (34).$$

where

$$\Delta_1=\frac{\left(\tau\delta^2+\sigma^2\right)\left(\delta_1^2+\pi^2\xi T_a\right)\begin{bmatrix}\delta_2{}^2-R_i-\lambda_2\sigma^2+\lambda_1\sigma^2\\ \left(1+\delta_2{}^2-R_i\right)\end{bmatrix}}{a^2\left(1+\lambda_1{}^2\sigma^2\right)}+\frac{a^2\left(1+\lambda_1\sigma^2\right)\tau Ra_s\left(\delta^2\tau\left(\delta_2{}^2-R_i\right)\sigma^2\right)}{\delta^2\tau+\sigma^2} \qquad (35)$$

$$\Delta_2=\frac{\left(\tau\delta^2+\sigma^2\right)\left(\delta_1^2+\pi^2\xi T_a\right)\left[\left(1+\delta_2{}^2-R_i\right)-\lambda_1\left(\delta_2{}^2-R_i-\lambda_2\sigma^2\right)\right]}{a^2\left(1+\lambda_1{}^2\sigma^2\right)}+\frac{a^2\left(1+\lambda_1\sigma^2\right)\tau Ra_s\left(\delta^2\tau-\left(\delta_2{}^2-R_i\right)\right)}{\delta^2\tau+\sigma^2}$$

For oscillatory mode $\Delta_2=0$ and $\sigma_i\neq 0$, where σ is the oscillatory frequency which is not given for brevity.
We have the necessary expression for oscillatory Rayleigh number as:

$$Ra_T^{osc}=\Delta_1 \qquad (36)$$

4. NONLINEAR STABILITY ANALYSIS

In this section, nonlinear stability has been studied using minimal truncated Fourier series. For simplicity, we consider only two dimensional rolls, so that all the physical quantities are independent of y. We introduce the stream function ψ as $u=\frac{\partial\psi}{\partial z}$, $w=-\frac{\partial\psi}{\partial x}$, then taking curl to eliminate pressure term from Eq.(2), to get

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)^2\left(\frac{\partial^2}{\partial x^2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\psi+\left(1+\lambda_1\frac{\partial}{\partial t}\right)^2\xi T_a\frac{\partial^2\psi}{\partial z^2}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left[\tau Ra_S\frac{\partial S}{\partial x}-Ra_T\frac{\partial T}{\partial x}\right] \qquad (37)$$

$$\left(\frac{\partial}{\partial t}-\left(\eta\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-R_i\right)\right)T-\frac{\partial(\psi,T)}{\partial(x,z)}+\frac{\partial\psi}{\partial x}=0 \qquad (38)$$

$$\left(\frac{\partial}{\partial t}-\tau\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\right)S-\frac{\partial(\psi,S)}{\partial(x,z)}+\frac{\partial\psi}{\partial x}=0 \qquad (39)$$

It is to be noted that the effect of nonlinearity is to distort the temperature and concentration fields through the interaction of ψ, with T and S. As a result a component of the form $Sin\,(2\pi z)$ will be generated. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi=M_0(t)\sin(ax)\sin(\pi z) \qquad (40)$$

$$T=M_1(t)\cos(ax)\sin(\pi z)+M_2(t)\sin(2\pi z) \qquad (41)$$

$$S=M_3(t)\cos(ax)\sin(\pi z)+M_4(t)\sin(2\pi z) \qquad (42)$$

where the amplitudes $M_0(t)$, $M_1(t)$, $M_2(t)$, $M_3(t)$, $M_4(t)$ are functions of time and are to be determined. Substituting above expressions in Eqs. (37) - (39) and equating the like terms, the following set of nonlinear autonomous differential equations is obtained

$$\frac{dM_0}{dt}=D_1(\mathrm{t}) \qquad (43)$$

$$\frac{dD_1}{dt}=-\frac{1}{\left(\lambda_2{}^2\delta_1{}^2+\pi^2\xi T_a\lambda_1{}^2\right)}\begin{bmatrix}\left(\delta_1^2+\pi^2\xi T_a\right)M_0+2\left(\lambda_1{}^2\delta_1{}^2+\pi^2\xi T_a\lambda_2{}^2\right)D_1\\ +aRa_TM_1-\tau aRa_SM_3+aRa_T\left(\lambda_1+\lambda_2\right)\frac{dM_1}{dt}\\ -\tau aRa_S\left(\lambda_1+\lambda_2\right)\frac{dM_3}{dt}+aRa_T\left(\lambda_1\lambda_2\right)\frac{d^2M_1}{dt^2}\\ -\tau aRa_S\left(\lambda_1\lambda_2\right)\frac{d^2M_3}{dt^2}\end{bmatrix} \qquad (44)$$

$$\frac{dM_1}{dt} = -\left[aM_0 + \pi a M_0 M_2 + \left(\delta_2^{\,2} - R_i\right)M_1\right] \quad (45)$$

$$\frac{dM_2}{dt} = -\left[\left(4\pi^2 - R_i\right)M_2 - \frac{\pi a}{2}M_0 M_1\right] \quad (46)$$

$$\frac{dM_3}{dt} = -\left[aM_0 + \pi a M_0 M_2 + \delta^2 \tau M_3\right] \quad (47)$$

$$\frac{dM_4}{dt} = -\left[4\pi^2 \tau M_4 - \frac{\pi a}{2}M_0 M_3\right] \quad (48)$$

Numerical method was used to solve the above nonlinear differential equation to find the amplitudes.

4.1 Steady Finite Amplitude Motions

For steady state finite amplitude convection, we have to set left hand side of the Eq. (43-48) equal to zero.

$$D_1(t) = 0 \quad (49)$$

$$\left(\delta_1^{\,2} + \pi^2 \xi T_a\right)M_0 + aRa_T M_1 - \tau a Ra_S M_3 = 0 \quad (50)$$

$$aM_0 + \pi a M_0 M_2 + \left(\delta_2^{\,2} - R_i\right)M_1 = 0 \quad (51)$$

$$\left(4\pi^2 - R_i\right)M_2 - \frac{\pi a}{2}M_0 M_1 = 0 \quad (52)$$

$$aM_0 + \pi a M_0 M_2 + \delta^2 \tau M_3 = 0 \quad (53)$$

$$4\pi^2 \tau M_4 - \frac{\pi a}{2}M_0 M_3 = 0 \quad (54)$$

On solving the above equations for the amplitudes, we obtain M_1, M_2, M_3, M_4 in terms of M_0 as

$$M_1 = -\frac{2a\left(4\pi^2 - R_i\right)M_0}{a^2 M_0^{\,2}\pi^2 + 2\left(4\pi^2 - R_i\right)\left(\delta_2^{\,2} - R_i\right)} \quad (55a)$$

$$M_2 = -\frac{a^2 \pi M_0^{\,2}}{a^2 M_0^{\,2}\pi^2 + 2\left(4\pi^2 - R_i\right)\left(\delta_2^{\,2} - R_i\right)} \quad (55b)$$

$$M_3 = -\frac{\left(8aM_0\tau\right)}{\left(a^2 M_0^{\,2} + 8\delta^2\tau^2\right)} \quad (55c)$$

$$M_4 = -\frac{\left(a^2 M_0 \tau\right)}{\pi\left(a^2 M_0^{\,2} + 8\delta^2\tau^2\right)} \quad (55d)$$

4.2 Steady Heat And Mass Transports

In the study of this type problem, quantification of heat and mass transport is very important in porous media. Let N_u and S_h be denoted as the rate of heat and mass transports per unit for the fluid phase known as Nusselt number and Sherwood number respectively, defined by

$$Nu = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial T}{\partial z}dx}{\int_0^{\frac{2\pi}{a}} \frac{\partial T_b}{\partial z}dx}\right]_{z=0} \quad (56)$$

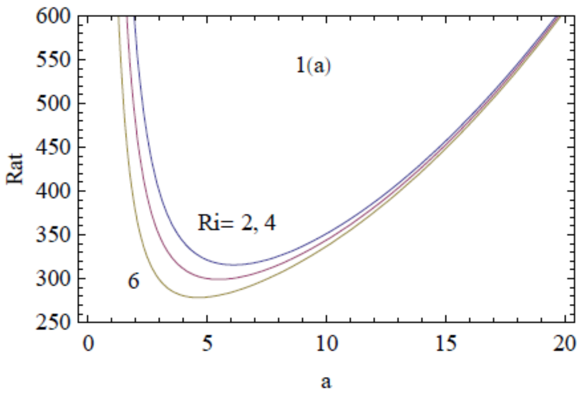
$$Sh = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial S}{\partial z}dx}{\int_0^{\frac{2\pi}{a}} \frac{\partial S_b}{\partial z}dx}\right]_{z=0}$$

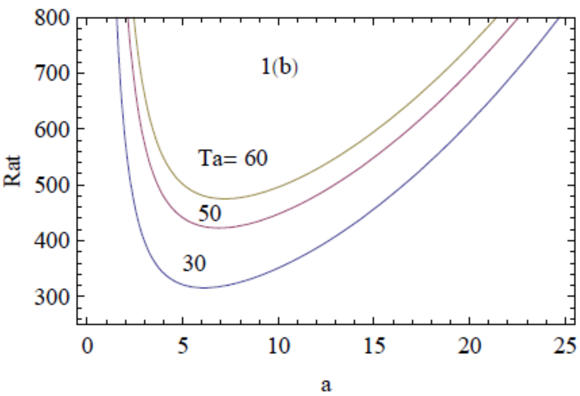
Substituting M_2, M_4 in (55a, 55b, 55c, 55d), the expressions for N_u and S_h are obtained as

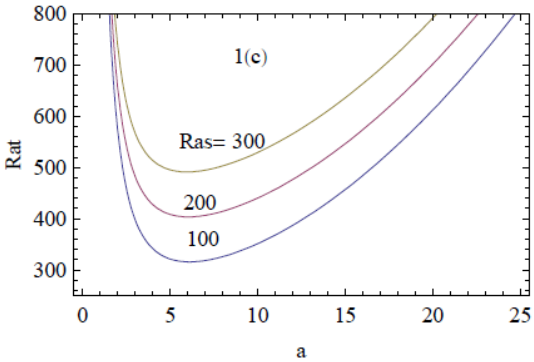
$$N_u = 1 - 2\pi M_2$$

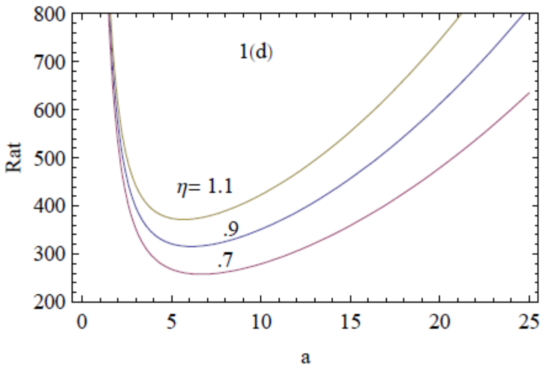
$$S_h = 1 - 2\pi M_4.$$

Figures









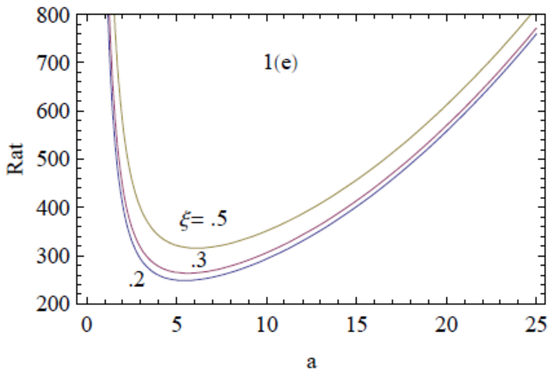
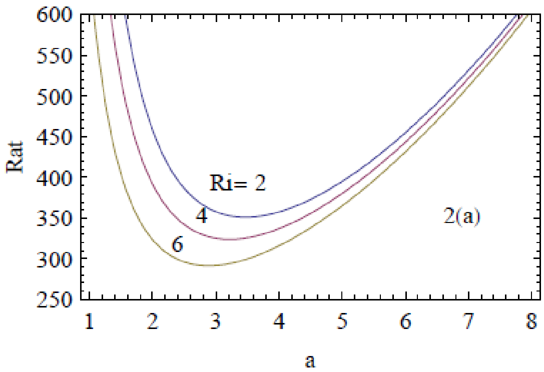
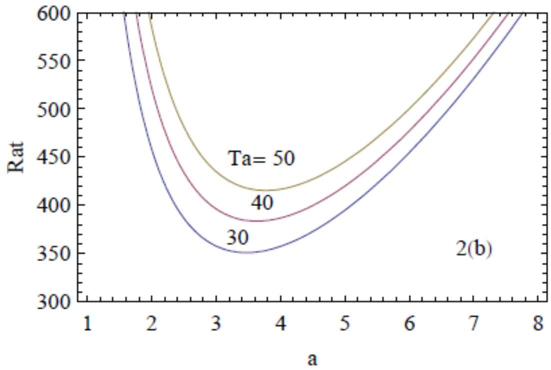
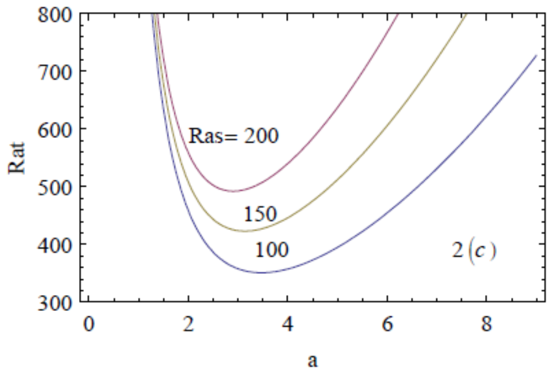
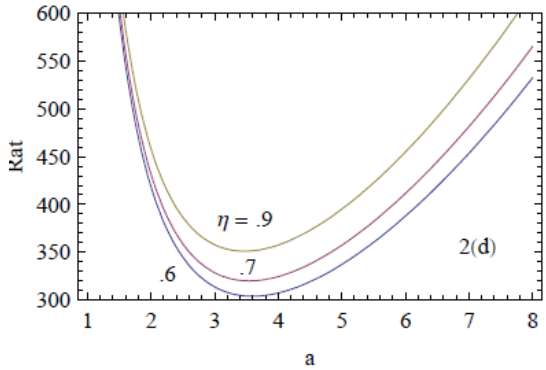


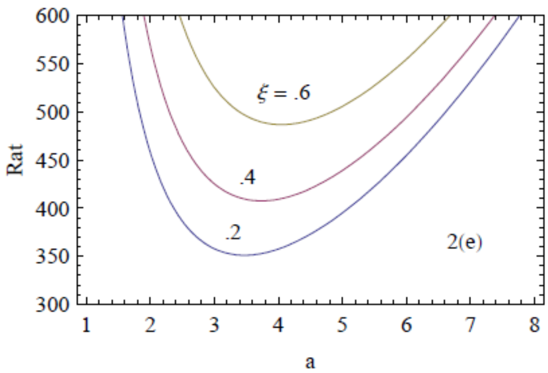
Fig1. Neutral stability curves for different values of different parameter

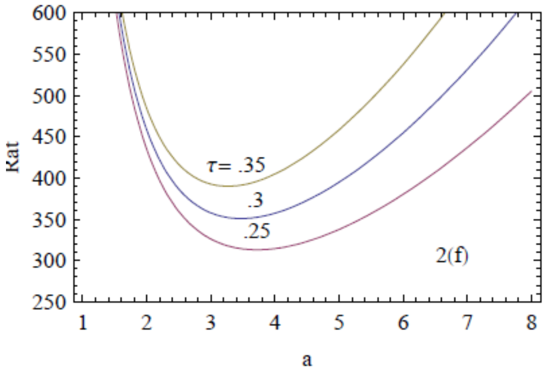


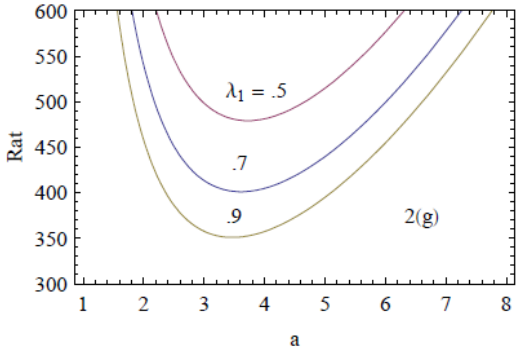












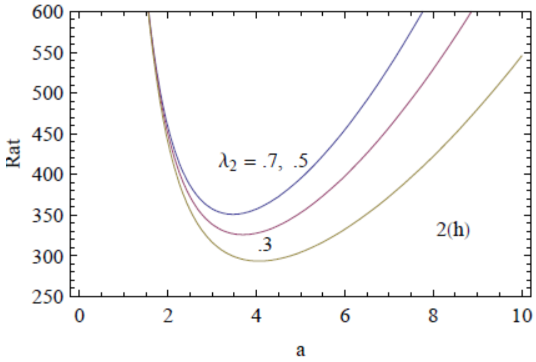
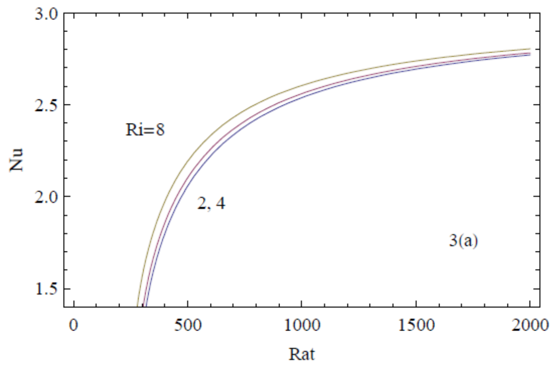
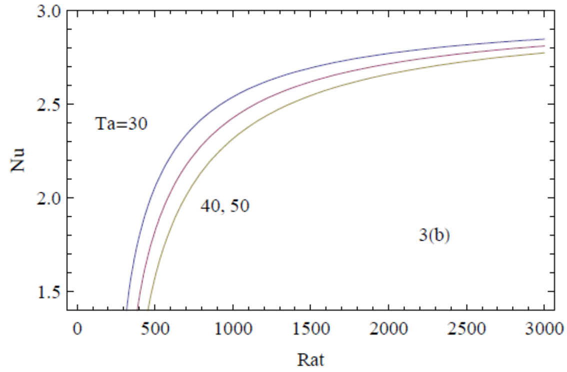
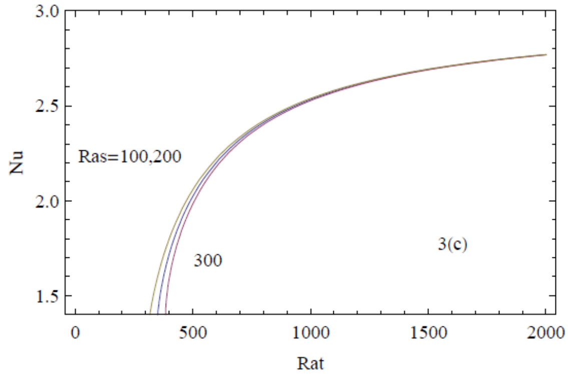
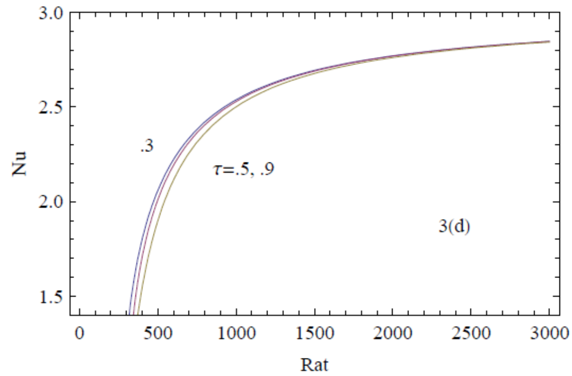


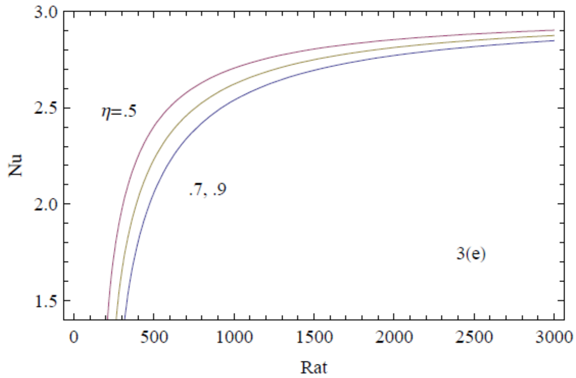
Fig 2. Ocsllatory stability curves for different values of different parameter











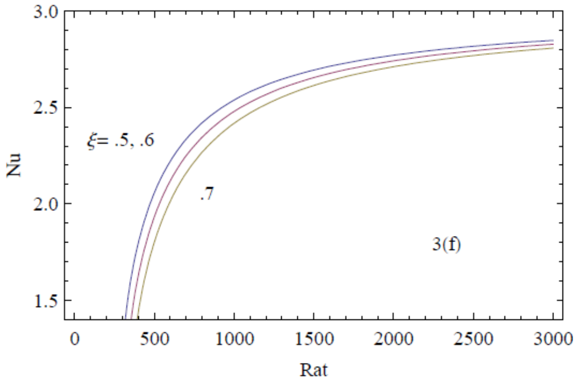
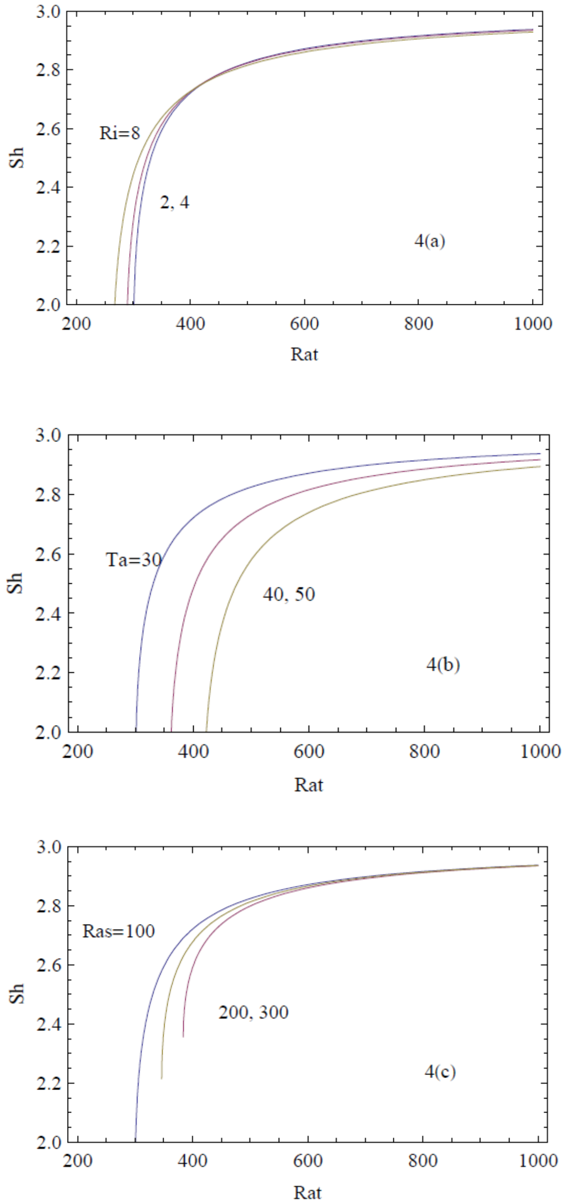


Fig 3.Nusselt number curves for different values of different parameter



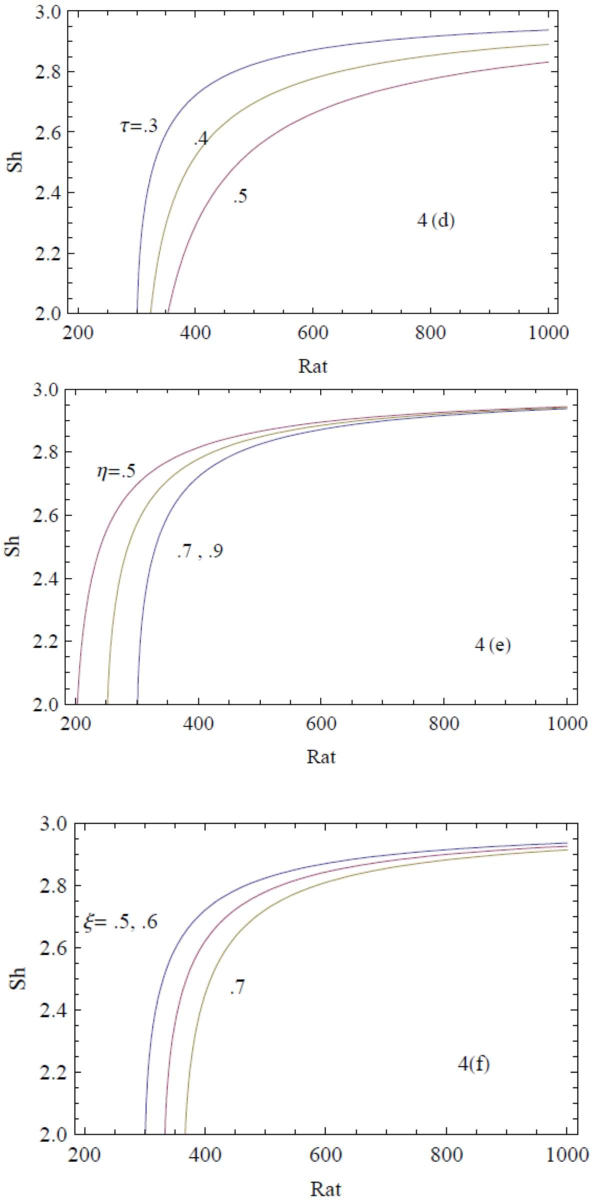


Fig 4.Sherwood number curves for different values of different parameter

5. RESULTS AND DISCUSSION

We have studied the effect of internal heat source on double diffusive convection in a viscoelastic fluid saturated rotating anisotropic porous layer using linear and nonlinear stability analyses. In this section, we obtain the effects of various parameters in the governing equations on the onset of double diffusive convection numerically and express them graphically. The numerical values of thermal Rayleigh number for stationary and oscillatory modes of convection for different values of the parameters such as Taylor number, relaxation and retardation parameters, solute

Rayleigh number, and parameter are computed, and depicted in figures.

Linear Stability

The marginal stability curves in the (Ra_T, a) plane for the stationary and oscillatory modes are presented through graphs for different values of the parameters. Figs 1(a-e) are for stationary mode, while Figs 2(a-h) correspond to oscillatory mode of convection. We fix the values for the parameters as $T_a =$ 30, $\xi = .5$, $Ra_S = 100$, $\tau = .3$, $\lambda_2 = .9$, $\lambda_1 = .5$, $\eta = .9$ and $R_i = 2$, except the varying parameter.

From Figs 1(a), 2(a), it is observed that on increasing the value of internal Rayleigh number R_i, the critical values of stationary and oscillatory Rayleigh number decrease, thus destabilizing the system. This shows that the effect of an increment in the value of R_i, is to advance the onset of both stationary as well as oscillatory modes of convection. However, from Figs 1(b), 2(b) for Taylor number T_a, Figs 1(c), 2(c) for solutal Rayleigh number Ra_S, Figs 1(d), 2(d) for thermal anisotropic parameter η and Figs 1(e), 2(e) for mechanical anisotropic parameter ξ, it is observed respectively that on increasing the values of T_a, Ra_S, η, ξ the critical values of stationary and oscillatory Rayleigh numbers increase, thus stabilizing the system. This shows that the effect of increasing the values of T_a, Ra_S, η, ξ is to delay the onset of stationary and oscillatory convection.

Further, it is found from Figs 2(f, h) that the effect of increasing the values of diffusivity ratio τ and the parameter λ_2 is to increase the critical value of the oscillatory Rayleigh number, thus delaying the onset of oscillatory convection. However opposite effect is found in Fig 2(g), where an increment in the value of parameter λ_1 decreases the critical value of the oscillatory Rayleigh number, thus advancing the onset of oscillatory convection.

Nonlinear Stability

The effects of various parameters on the rate of heat and mass transfer are shown in Fig 3 and Fig 4 respectively. Figs 3(a) and 4(a) show that an increment in the value of the internal Rayleigh number R_i increases the values of both Nusselt number N_u and Sherwood number S_h, which is due to the fact that increasing the value of R_i advances the onset of convection. From Figs 3(b) and 4(b) for Taylor number T_a, Figs 3(c), 4(c) for solute Rayleigh number Ra_S, Figs 3(d), 4(d) for diffusivity ratio τ, Figs 3(e), 4(e) for thermal anisotropic parameter η, Figs 3(f), 4(f) for mechanical anisotropic parameter ξ, it is observed that on increasing the values of T_a, Ra_S, τ, η and ξ, the values of both Nusselt number N_u and Sherwood number S_h decrease, thus stabilizing the system.

6. CONCLUSIONS

In this paper, internal heating effect on double diffusive convection in a viscoelastic fluid saturated rotating anisotropic porous layer, which is heated and salted from below, is investigated. The problem has been solved analytically, performing linear and nonlinear analyses. Linear analysis is done using normal mode technique. Following conclusions are drawn:

1) The Taylor number T_a, mechanical anisotropic parameter ξ, solute Rayleigh number Ra_S and thermal anisotropic parameter η has a stabilizing effect on the both stationary and oscillatory convection.
2) The internal heat parameter R_i destabilizes the system in the stationary and oscillatory system.
3) The effects of diffusivity ratio τ and retardation parameter λ_2 have stabilizing effect on the oscillatory convection.
4) The relaxation parameter λ_1 has a destabilizing effect on the oscillatory convection.
5) The increasing the value of internal Rayleigh number R_i then increase the value of Nusselt number N_u i.e. increased heat transfer but increasing the value of mechanical anisotropic parameter ξ, Taylor number T_a, solute Rayleigh number Ra_S, diffusivity ratio τ and thermal anisotropic parameter η decreases the value ofNusselt number N_u.
6) Mass transfer that is the value of Sherwood number increases on increasing the value of internal Rayleigh number R_i, while decreaseson increasing the values of mechanical anisotropic parameter ξ, Taylor

number T_a ,solute Rayleigh number Ra_S, diffusivity ratio τ and thermal anisotropic parameter η.

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Appendix

Latin symbols

a	wave number
d	depth of porous layer
g	Acceleration due to gravity
τ	Diffusivity ratio $\tau = \frac{\kappa_S}{\kappa_{Tz}}$
Ra_T	Thermal Rayleigh number $Ra_T = \frac{\beta_T g \Delta T K_z d}{\nu \kappa_{Tz}}$
Ra_S	Solute Rayleigh number $Ra_S = \frac{\beta_S g \Delta S K_z d}{\nu \kappa_{Tz}}$
K	permeability
T	temperature
S	solute concentration
ΔT	Temperature difference across the porous layer
ΔS	Solute difference across the porous layer
t	time
p	reduced pressure
q	Fluid velocity (u,v,w)
Pr_D	Prandtl number $Pr_D = \frac{\phi \gamma \nu d^2}{\kappa_T k}$
R_i	Internal Rayleigh number $R_i = \frac{Qd^2}{\kappa_T}$
T_a	Taylor number $T_a = \left(\frac{2\Omega K_z}{\mu\phi}\right)^2$
Q	Internal heat source
N_u	Nusselt number

S_h	Sherwood number
(x,y,z)	Space co-ordinates

Greek symbols

κ_T	Effective thermal diffusivity $\kappa_{Tx}\left(ii+jj\right)+\kappa_{Tz}\left(kk\right)$
κ_{Tx}	Effective thermal diffusivity in x-direction
κ_{Tz}	Effective thermal diffusivity in direction
β_T	Coefficient of thermal expansion
β_S	Coefficient of solute expansion
$\overline{\lambda_1}$	Stress-relaxation time
$\overline{\lambda_2}$	Strain-retardation time
λ_1	Relaxation parameter $\left(\frac{\kappa_{Tz}}{\gamma d^2}\right)\overline{\lambda_1}$
λ_2	Retardation parameter $\left(\frac{\kappa_{Tz}}{\gamma d^2}\right)\overline{\lambda_2}$
T_0	Reference temperature
S_0	Reference concentration
σ	Growth rate
μ	Dynamic viscosity of the fluid
μ_c	Effective viscosity of the fluid
ϕ	Porosity
γ	Heat capacities ratio $\frac{\left(\rho c_p\right)_m}{\left(\rho c_p\right)_f}$
ν	Kinematic viscosity $\frac{\mu}{\rho_0}$
ρ	Fluid density
ρ_0	Reference density

Other symbols

∇_1^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$
∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

Subscripts

b	basic state
c	critical
0	reference value

Superscripts

'	perturbed quantity
*	Dimensionless quantity
osc	oscillatory
st	stationary